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Consistency of higher derivative gravity in the brane background

Ishwaree P. Neupane

Department of Physics, Seoul National University
Seoul, 151-742, Korea
E-mail: `ishwaree@phya.snu.ac.kr`

Abstract: We consider the theory of higher derivative gravity with non-factorizable Randall-Sundrum type space-time and obtain the metric solutions which characterize the p-brane world-volume as a curved or planar defect embedded in the higher dimensions. We consider the string inspired effective actions of the dilatonic Gauss-Bonnet type in the brane background and show its consistency with the RS brane-world scenario and the conformal weights of the dilaton couplings in the string theory with the appropriate choice of Regge slope (α') or Gauss-Bonnet coupling (α) or both. We also discuss a time dependent dilaton solution for a string-inspired fourth-derivative gravity.

Keywords: p-branes, Brane Dynamics in Gauge Theories, Classical Theories of Gravity, Models of Quantum Gravity.

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Contents

1. Introduction

Initially, the higher-dimensional gravity theories [1, 2] were realized with the assumptions that the realistic Kaluza-Klein theories [3] require compact extra spatial dimensions, other than $1+3$ space-time world, of the order of Planck lengths. However, this idea has got a new interest and direction after the intriguing ideas of Randall and Sundrun (RS) that the physical $(3+1)$-dimensional space-time world can be embedded as a hypersurface in a higher-dimensional Anti-deSitter (AdS) bulk space-time [4], leaving the extra non-compact spatial dimensions [5] of the size of mm or even infinite [4, 6]. What is observed crucial about this brane scenario is that at shorter distance than the AdS curvature length, the extra dimensions reveal to change the Einstein gravity significantly and above the AdS radius it is effectively approximated by Einstein's gravity in $3+1$ space-time dimension [4, 7], pointing the fact that the correction is much suppressed as the number of transverse extra dimensions grow [8]. This has left room for the gravity (and possibly the spin 0 counterpart of graviton, i.e. dilaton) to flow freely in the bulk, whereas the standard matter particles are effectively confined in the brane. Such localization of matter on the brane [4, 8, 9] has changed our conventional wisdom on KK theories [3], i.e. the size of extra dimensions does not have to be $1/M_{\rm pl} \sim 10^{-33}\,{\rm cm}$, but could be much bigger.

In most of recent work [8]–[14] to the solution of Einstein equations in $n \geq 1$ extra dimensions, the $3+1$ dimensions of our world is identified with the internal space of topological defects residing in a higher-dimensional space-time. Specifically, the 3-brane world volume is pictured as a domain wall propagating in the five-dimensional bulk space-time [4, 9], as a local string defect residing in the six-dimensional space-time [8]–[11], as a global monopole defect with a conical deficit angle in seven-dimensional space-time [14] and its continuation with the co-dimensions four characterizes the instanton solution which describes the quantum nucleation of a five-dimensional brane-world [13, 14]. Most of these and similar work have been judged only from the solutions to Einstein gravity in the extra dimensions. Also in [15] an idea about the creation of a spherically symmetric brane-world for a 1-brane and a 3-brane in the context of Wheeler-De Witt equation and WKB approximation is furnished. Thus, in this paper, we shall find metric solutions that also include the contribution from higher-curvature terms in the field equations and describe the p-brane object as a curved or planar defect residing in the higher dimensions.

In order to check the compatibility of the various string inspired low energy effective actions under the brane scenarios and to realize the very natural and more fundamental theory of gravity one cannot avoid the higher order curvature terms in the effective action. This is so from the different perspectives such as the renormalizibility and the asymptotic freedom of the theory in 4 dimensions [16], and for the stabilization of the scalar potential [2] in the bulk. Also the action containing second powers of the curvature tensors naturally arise in the string effective action [17]. This has further attraction with the hope that some versions of the higher derivative gravity represent the supergravity duals when R and Λ are the leading order and higher derivatives are of next to leading order in large-N expansion (see [18] and references therein). The effective action containing just R cannot stabilize the scalar potential defined in the bulk, otherwise, the bulk gauge sector appears in conflict with the large extra dimensions [2, 6].

An investigation of the higher derivative gravity by adding a dilatonic potential to the effective action is made in ref. [19] where a naked singularity in AdS_5 bulk persists for the finite $(3+1)$-dimensional Planck mass and dilaton potential acts as the bulk cosmological constant. Also various static and inflationary solutions of the R-S model in Gauss-Bonnet combination have been studied in [20]. But, what is missing in these models is either dilaton field or the proper coupling of dilaton field to the higher-curvature terms or both. So we consider the string inspired effective action [21, 22, 23] in dilatonic Gauss-Bonnet combination taking the proper care of conformal weights of the dilaton couplings. A work along this line is recently reported in [24].

The paper is organized as follows. In the next section we give the solutions to higher derivative gravity in the background of Ricci constant curvature space-time. In section 3 we derive the Einstein field equations for a non-factorizable general metric ansatz and will be subsequently utilized in section 4 to obtain the metric solutions corresponding to higher-dimensional defects. In section 5 we also include the contribution of the higher curvature terms to the field equations and find the metric solutions characterizing planar wall and planar string defects. In section 6 we consider string inspired effective actions in dilatonic Gauss-Bonnet combination and study the compatibility of the theory with RS type space-time and the conformal weights of the dilaton couplings derived from the heterotic type-I string theory in five space-time dimensions. Our discussions and outlook of the problems are summarized in section 7.

2. Higher derivative gravity

The most general effective action involving invariants with mass dimension ≥ 4 and vanishing torsion and dilaton field for the higher derivative gravity is given by

$$S = \int d^{d+1}x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \Lambda_b + \lambda'\left(\alpha R^2 + \beta R_{AB}R^{AB} + \gamma R_{ABCD}R^{ABCD}\right)\right\} + \\ + S_B + S_m\,, \tag{2.1}$$

where Λ_b is the bulk cosmological constant, S_B is the boundary action and S_m is the matter action which may include the matter trapped in the p-brane and the $D(= d+1)$-dimensional mass term is defined by $\kappa^2 = 8\pi G_{d+1} = 8\pi M^{1-d}$. Here we set the string coupling $\lambda' = \alpha'/8g_s^2 = 1$ for simplicity and but the effective action with the proper couplings of dilaton field to the higher order curvature terms will be presented in section 6.

The classical field equations derived by varying the above action w.r.to g^{AB} can then be expressed in the form

$$G_{AB} + \kappa^2 X_{AB} = -\kappa^2(\Lambda_b g_{AB} - T_{AB})\,, \tag{2.2}$$

where the energy-momentum tensor is defined in the well-known form

$$T_{AB} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{AB}} = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{AB}}\int d^{d+1}x\sqrt{-g}\,\mathcal{L}_m\,, \tag{2.3}$$

$G_{AB} = R_{AB} - g_{AB}R/2$ and X_{AB} is given by

$$\begin{aligned} X_{AB} = &-\frac{1}{2}g_{AB}(\alpha R^2 + \beta R_{CD}R^{CD} + \gamma R_{CDEF}R^{CDEF}) + \\ &+ 2[\alpha R R_{AB} + \beta R_{ACBD}R^{CD} + \gamma(R_{ACDE}R_B{}^{CDE} - 2R_A{}^C R_{BC} + \\ &\quad + 2R_{ACBD}R^{CD})] - (2\alpha + \beta + 2\gamma)(\nabla_A\nabla_B R - g_{AB}\nabla^2 R) + \\ &+ (\beta + 4\gamma)\nabla^2 G_{AB}\,, \end{aligned} \tag{2.4}$$

where $A, B, \ldots$ denote the $D(= d + 1)$-dimensional space-time indices and the signature we follow is $(-, +, +, +, \ldots)$. The last line of the above expression vanishes for two cases, i.e. (i) for the Gauss-Bonnet relation (i.e. $4\alpha = -\beta = 4\gamma$) and (ii) for the background metric of Ricci constant curvature space-time (in which case, since $R_{AB} \propto g_{AB}$ and the curvatures are covariantly constant). To simplify the calculation to a greater extent, we want to keep the freedom in the choice of relations among α, β and γ for the later purpose and mainly consider the second case in what follows below in this section. We briefly review the general features of the action (2.1) at first. The general curvature squared terms in the d spatial dimensions can be written as [23],

$$\begin{aligned} &\alpha R^2 + \beta R_{AB}R^{AB} + \gamma R_{ABCD}R^{ABCD} = \\ &\quad = -\left[\frac{(d-1)\beta + 4\gamma}{4(d-2)}\right]\mathcal{R}^2_{GB} + \left(\frac{d-1}{d-2}\right)(\beta/4 + \gamma)C^2 + \\ &\qquad + \left[\frac{4d\alpha + (d+1)\beta + 4\gamma}{4d}\right]R^2\,, \end{aligned} \tag{2.5}$$

where the Gauss-Bonnet term ($\mathcal{R}^2_{GB}$) and the square of the Weyl tensor (C^2) are given by

$$\begin{aligned} \mathcal{R}^2_{GB} &= R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\,, \\ C^2 &= \frac{2}{d(d-1)}R^2 - \frac{4}{d-1}R^2_{AB} + R_{ABCD}R^{ABCD}\,. \end{aligned} \tag{2.6}$$

Of course, for $d = 3$, $\mathcal{R}^2_{GB}$ is simply the integrand of the Gauss-Bonnet term. Since the Weyl tensor vanishes for a conformally flat AdS metric, with the requirement that $16\alpha + 5\beta + 4\gamma = 0$ for $d = 4$, the curvature squared terms naturally arise in the Gauss-Bonnet combination. In the scheme where R^2_{ABCD} can be modified to C^2_{ABCD} the action (2.1) may be deduced from the heterotic string via heterotic type-I duality [21, 22]. Furthermore, the theory with the Weyl term squared corresponds to the setting $\alpha = -\beta/2d = 2\gamma/d(d-1)$. For $d = 4$ this means $\alpha = \gamma/6$, $\beta = -4\gamma/3$ and; evidently, the higher derivative part of the above action is reduced to

$$S = \int d^{d+1}x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \Lambda_b + \gamma C_{ABCD}C^{ABCD}\right) + S_m + S_B\,. \tag{2.7}$$

With $T_{AB} = 0$, the theory defined by the above action (2.7) can have only the flat space-time as the vacuum solution. Forthermore, the Einstein gravity with the cosmological constant term is characterized by the setting $\alpha = \beta = \gamma = 0$, i.e.

$$S = \int d^{d+1}x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \Lambda_b\right) + \int d^dx\sqrt{-g}\mathcal{L}_m + S_B\,. \tag{2.8}$$

This gives (with $S_B = 0$ and $T_{AB} = 0$) $R_{AB} - g_{AB}R/2 = -\kappa^2\Lambda_b g_{AB}$ and the corresponding $(d+1)$-dimensional Ricci curvature and Ricci scalar are then given by

$$R_{AB} = -\frac{d}{l^2}g_{AB}\,, \qquad R = -\frac{d(d+1)}{l^2}\,, \tag{2.9}$$

where the square of the curvature radius is defined by $l^2 = -d(d-1)/2\kappa^2\Lambda_b$. For $d = 3$, this is the square of the scalar curvature radius in the Minkowski space-time. In our notations AdS(dS) solution corresponds to $\Lambda_b < 0$, $l^2 > 0$ ($\Lambda_b > 0$, $l^2 < 0$). We assume that, for $d + 1 = 5$, the equations of motion derived from (2.1) can have a solution describing the AdS_5 spacetime. This is indeed the case and whose metric solution is given by [18],

$$ds^2 = \frac{1}{r}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{l^2}{4r^2}dr^2\,. \tag{2.10}$$

This is actually a conformally flat AdS metric solution. For AdS_5, the scalar, Ricci and Riemann curvatures (derived from the metric (2.10) or eq. (2.9)) are given by ($l^2 > 0$),

$$\begin{aligned} R &= -\frac{20}{l^2}\,, \qquad R_{AB} = -\frac{4}{l^2}g_{AB}\,, \\ R_{ABCD} &= -\frac{1}{l^2}(g_{AC}g_{BD} - g_{AD}g_{BC})\,, \quad l^2 > 0\,. \end{aligned} \tag{2.11}$$

This simply implies that the vacuum solution of Einstein gravity is equivalent to the higher derivative gravity theory with constant Riemann curvature bulk AdS. One then easily evaluates

$$R_{ACDE}R_B{}^{CDE} = -\frac{2d}{l^4}g_{AB}\,, \qquad R_{ABCD}R^{ABCD} = -\frac{2d(d+1)}{l^4}\,. \tag{2.12}$$

This shows that space-time with constant Riemann curvature characterizes the AdS_{d+1} with $\Lambda_b < 0$ and $l^2 > 0$. The relations (2.11) and (2.12) are equally applied to the $(d+1)$-dimensional conformally flat metric solution given by

$$ds^2 = \frac{1}{r}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{l^2}{4r^2}dr^2 + \frac{1}{r}d^{n-1}X^2\,, \tag{2.13}$$

where $d^{n-1}X^2$ denotes the $(n-1)$-dimensional flat space metric. With the above subtitutions into eq. (2.2) with $T_{AB}=0$, the field equations reduce to the following form:

$$\frac{d(d-1)}{2\kappa^2 l^2} - \frac{d(d-3)}{l^4}[d(d+1)\alpha + d\beta + 2\gamma] + \Lambda_b = 0\,. \tag{2.14}$$

There exists two classes of general solutions of eq. (2.14), i.e. (i) for $l^2 \to \infty$: the solution is a flat Minkowski space-time with $\Lambda_b \to 0$ and (ii) for the vanishing bulk cosmological constant, $(\Lambda_b = 0)$, one has

$$l^2 = \frac{2(d-3)}{d-1}[d(d+1)\alpha + d\beta + 2\gamma]\kappa^2\,, \tag{2.15}$$

therefore, for $d > 3$, there exist AdS(dS) solution if the quantity in the parenthesis is positive (negative). However for other values of d, viz $d \le 3$, the case is less interesting. Specifically, the curvature diverges for $d = 3$ and $d = 1$ characterizes a flat space solution. For $d = 2$, the space-time would be dS (AdS) when the quantity in the parenthesis is positive (negative). Equation (2.14) is solved for l^2 if,

$$\left(\frac{d(d-1)}{2\kappa^2}\right)^2 + 4d(d-3)\left[d(d+1)\alpha + d\beta + 2\gamma\right]\Lambda_b > 0\,. \tag{2.16}$$

A connection of this result to the Supergravity dual of $\mathcal{N} = 2\,\mathrm{Sp}(N)$ and superconformal field theory is discussed in [18]. With the Gauss-Bonnet relation, i.e. $\alpha = -\beta/4 = \gamma$, eq. (2.16) implies

$$\alpha > -\frac{d(d-1)}{16\Lambda_b(d-2)(d-3)}\frac{1}{\kappa^4}\,. \tag{2.17}$$

This implies that for $d > 3$ and $\alpha > 0$, the bulk space-time is always AdS. And for the free parameters α, β, γ, the space-time can be guaranted as AdS_{d+1} if the following inequality holds,

$$4\Lambda_b d(d-3)[d(d+1)\alpha + d\beta + 2\gamma] < 0\,. \tag{2.18}$$

3. Einstein field equations

In this section we consider a general non-factorizable metric ansatz which might respect the $4d$ Poincaré invariance. More specifically, we consider the the presence of the brane and gravity. Due to the gravitational field on the brane, the brane can be curved and this would eventually rise to give a non-zero cosmological constant in $3+1$ space-time dimensions. A $(d+1)$-dimensional metric ansatz satisfying this assumption is

$$\begin{aligned} ds^2 &= g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu + \gamma_{ij}dx^i dx^j \\ &= e^{-2M(r)}\hat{g}_{\mu\nu}dx^\mu dx^\nu + e^{-2N(r)}(dr^2 + d\Omega_{n-1}^2)\,, \end{aligned} \tag{3.1}$$

where $\mu, \nu, \ldots$ denote the space-time indices of the p-brane, and $i, j, \ldots$ denote n-dimensional extra spatial indices. We also parameterize the $(n-1)$-dimensional sphere recursively as

$$d\Omega^2_{n-1} = d\theta^2_{n-1} + \sin^2\theta_{n-1} d\Omega^2_{n-2} \,. \tag{3.2}$$

So the equality $d = p + n$ holds and in this paper we concentrate on the case of $p \geq 3$. For simplicity one can reparametrize the ansatz with the transformation $dy = e^{-N(r)} dr$ so that

$$ds^2 = e^{-2\,\mathrm{U}(y)} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dy^2 + e^{-2V(y)} d\Omega^2_{n-1} \,, \tag{3.3}$$

where y is the bulk coordinate. Ofcourse, for $d{+}1 = 5$ and $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$, $\mathrm{U}(y) = (1{+}k|y|)$ would explain the RS type solution, where k has the inverse dimension of AdS_5 curvature radius. Variation of the action (2.8) with respect to the $(d{+}1)$-dimensional metric tensor g_{AB}, with $S_B = 0$, leads to the Einstein's equations,

$$R_{AB} - \frac{1}{2} g_{AB} R = -\kappa^2 \left(\Lambda_b g_{AB} - T_{AB}\right). \tag{3.4}$$

Here, we are interested to consider a spherically symmetric ansatz for the energy-momentum tensor in the form

$$T^\mu_\nu = \delta^\mu_\nu \rho_o(y) \,, \qquad T^r_r = \rho_y(y) \,, \qquad T^{\theta_1}_{\theta_1} = T^{\theta_2}_{\theta_2} = \cdots = T^{\theta_{n-1}}_{\theta_{n-1}} = \rho_\theta(y) \,, \tag{3.5}$$

where $\rho_i (i = 0, y, \theta)$ are the brane sources defined in a general $(d+1)$-dimensional space-time. With the metric ansatz (3.3), we evaluate the followings non-trivial components of the Einstein tensor,

$$\begin{aligned} G_t{}^t &= p\Big[(n-1)U'V' - U'' + \frac{(p+1)}{2}(U')^2\Big] - \frac{n-1}{2}\Big[2V'' - n(V')^2 + (n-2)e^{2V}\Big] - \\ &\quad - \Lambda_p e^{2U} \,, \end{aligned} \tag{3.6}$$

$$\begin{aligned} G_y{}^y &= \frac{p+1}{2}\Big[2(n-1)U'V' + p(U')^2\Big] + \frac{(n-1)(n-2)}{2}\Big[(V')^2 - e^{2V}\Big] - \\ &\quad - \frac{p+1}{p-1}\Lambda_p e^{2U} \,, \end{aligned} \tag{3.7}$$

$$\begin{aligned} G_\theta{}^\theta &= \frac{p+1}{2}\Big[(p+2)(U')^2 - 2U'' + 2(n-2)U'V'\Big] - \frac{n-2}{2}\Big[2V'' - (n-1)(V')^2\Big] - \\ &\quad - \frac{(n-2)(n-3)}{2}e^{2V} - \frac{p+1}{p-1}\Lambda_p e^{2U} \,. \end{aligned} \tag{3.8}$$

In general, the Ricci scalar corresponding to the metric ansatz (3.3) is

$$\begin{aligned} R = {} & \frac{2(p+1)}{p-1}\Lambda_p e^{2U} + (p+1)(2U'' - (p+2)(U')^2 - 2(n-1)U'V') + \\ & + (n-1)\{2V'' - n(V')^2 + (n-2)e^{2V}\} \,, \end{aligned} \tag{3.9}$$

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where the $'$ denotes the differentiation d/dy. In deriving the above equations we have utilized the $(p+1)$-dimensional Ricci scalar, $\hat{R} = 2\Lambda_p(p+1)/(p-1)$ obtained from the $(p+1)$-dimensional Einstein equation

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = -\Lambda_p \hat{g}_{\mu\nu} \,. \tag{3.10}$$

Here we define the cosmological constant on the p-brane by the relation $\Lambda_p = 8\pi\Lambda_{\rm phy} M_{\rm pl}^{p-1}$, and $\Lambda_{\rm phy}$ is the physical cosmological constant defined in usual $(3+1)$ space-time. By eliminating two of the above equations or equivalently by using the energy momentum conservation law, $\nabla^A T_{AB} = 0$, we get

$$(p+1)(\rho_y - \rho_0)U' + (n-1)(\rho_y - \rho_\theta)V' = \rho_y' \,, \tag{3.11}$$

which is just the $(d+1)$-dimensional Bianchi identity derived from the Einstein field equations, i.e., after substituting (3.6)–(3.8) and (3.5) into (3.10).

4. Global defect solutions

For the completeness we briefly discuss on the Eintein gravity (i.e. $\alpha = \beta = \gamma = 0$) in higher dimensions, $n \geq 1$, and identify the p-brane world volume as the internal space of topological defect residing in the higher-dimensional space-time.

4.1 Domain wall solution

For the codimension one, $n = 1$, the Einstein field equations (3.4) with (3.6)–(3.8), and the ansatz (3.5), reduce to the following form:

$$2pU'' - p(p+1)(U')^2 = 2\kappa^2[\Lambda_b - \rho_0(y)] - 2\Lambda_p e^{2U} \,, \tag{4.1}$$

$$-p(p+1)(U')^2 = 2\kappa^2[\Lambda_b - \rho_y(y)] - \frac{2(p+1)}{p-1}\Lambda_p e^{2U} \,, \tag{4.2}$$

and the Bianchi identity (3.11) turns to the simpler form

$$\rho_y' = (p+1)(\rho_y - \rho_0)U' \,. \tag{4.3}$$

Now we consider the geometry having a warp factor, that is, $\mathrm{U}(y) = u|y|$, where u is a constant. This ensures the localization of gravity to the p-brane. With this ansatz, one easily solves eqs. (4.1) and (4.2) to get the following general metric solution [4, 9]

$$ds^2 = e^{-2u|y|}\hat{g}_{\mu\nu}dx^\mu dx^\nu + dy^2 \,, \tag{4.4}$$

where

$$\hat{R}_{(p+1)} = (p+1)\kappa^2(\rho_0 - \rho_y)e^{-2u|y|} \,, \tag{4.5}$$

$$u^2 = \frac{\kappa^2}{p(p+1)}[(p+1)\rho_0 + (p-1)\rho_y - 2\Lambda_b] \,. \tag{4.6}$$

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For the physically interesting case of 3-brane, one has the following general solutions:

$$\begin{aligned}\hat{R}_{(4)} &= 4\Lambda_p = 4\kappa^2(\rho_0 - \rho_y)e^{-2u|y|}\,,\\ u^2 &= \frac{\kappa^2}{6}(2\rho_0 - \rho_y - \Lambda_b)\,.\end{aligned} \tag{4.7}$$

There exists two special special cases of these general solutions: i.e.

(i) for the trivial sources in the extra dimensions (i.e. $\rho_0 = \rho_y = 0$), one has

$$\hat{R}_{(4)} = \frac{32\pi\Lambda_{\text{phy}}}{M_{\text{pl}}^2} = 0\,, \qquad u = \sqrt{-\frac{4\pi\Lambda_b}{3M_{(5)}^3}} \tag{4.8}$$

and hence the solution with $\Lambda_b < 0$ ensures the bulk geometry as AdS_5 and the brane geometry as Ricci flat with $\Lambda_{\text{phy}} = 0$, and

(ii) the constant source terms in the extra dimensions (i.e. $\rho_0 = \rho_y = \text{const}$) correspond to the spontaneous symmetry breaking in the extra dimensions.

For the second case one has

$$\hat{R}_{(4)} = 0\,, \qquad u = \sqrt{\frac{4\pi(\rho_0 - \Lambda_b)}{3M_{(5)}^3}}\,. \tag{4.9}$$

Clearly, $\rho_0 > \Lambda_b$ guarantees the positivity of u and exponentially decreasing warp factor needed to explain the hierarchy between electroweak scale and Planck scale mass [6, 4] and the fine tuning $\rho_0 = \Lambda_b$ characterizes a flat 5d Minkowski space-time solution.

4.2 String-like solution

For the case of $n = 2$ the field equations reduce to the form

$$\begin{aligned}V'' - (V')^2 - puV' - \frac{p(p+1)}{2}u^2 &= \kappa^2(\Lambda_b - \rho_0) - \Lambda_p e^{2u|y|}\,,\\ -(p+1)uV' - \frac{p(p+1)}{2}u^2 &= \kappa^2(\Lambda_b - \rho_y) - \frac{(p+1)}{(p-1)}\Lambda_p e^{2u|y|}\,,\\ -\frac{(p+1)(p+2)}{2}u^2 &= \kappa^2(\Lambda_b - \rho_\theta) - \frac{(p+1)}{(p-1)}\Lambda_p e^{2u|y|}\,,\end{aligned} \tag{4.10}$$

and the Bianchi identity (3.11) yields

$$\rho_y' = u(p+1)(\rho_y - \rho_0) + (\rho_y - \rho_\theta)V'\,. \tag{4.11}$$

Solving the above field equations, the general metric solution can be expressed in the form

$$ds^2 = e^{-2u|y|}\hat{g}_{\mu\nu}dx^\mu dx^\nu + dy^2 + e^{-2V(y)}d\theta^2\,, \tag{4.12}$$

where

$$
\begin{aligned}
u^2 &= \frac{1}{2(p+1)(p+2)}\left\{4\kappa^2(\rho_\theta-\Lambda_b)+\hat{R}_{(4)}e^{2u|y|}\right\},\\
V(y) &= uy+\frac{\kappa^2}{u(p+1)}\int dy(\rho_y-\rho_\theta)\,,\\
\hat{R}_{(4)} &= \frac{\kappa^2}{u}(\alpha-\rho_\theta)e^{-2u|y|}\,,
\end{aligned}
\tag{4.13}
$$

where α is an integration constant. Notice that the last equation above demands a definite form for ρ_θ. One crucial observation is that the inequality $\alpha > \rho_\theta$ keeps $\hat{R}_{(4)}$ positive and $(\rho_\theta - \Lambda_b) \geq 0$ ensures the positivity of u^2. Obviously, unlike the case of 3-brane in domain wall solution, where only the case $\Lambda_b < 0$ ensures $u^2 > 0$, for the string like solution one can either have $+ve$ or $-ve$ bulk cosmological constant to guarantee $u^2 > 0$ if $(\rho_\theta - \Lambda_b) > 0$ still holds. It is also reported in [9] that for the trivial sources in the extra dimensions, (i.e. $\rho_i = 0$), only $\Lambda_b < 0$ can ensure $u^2 > 0$. But this is not always the case as long as we do not set the integration constant $\alpha = 0$. As a specific solution as $-\rho_y = \rho_\theta = \alpha =$ constant, spontaneous symmetry breaking condition in the extra dimensions [12, 9], one obtains

$$
ds^2 = e^{-2u|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 + r_0^2 e^{-2u_0|y|}d\theta^2\,,
\tag{4.14}
$$

where

$$
u^2 = \frac{2\kappa^2}{(p+1)(p+2)}(\rho_\theta-\Lambda_b) > 0\,, \qquad u_0 = u - \frac{2\kappa^2\rho_\theta}{u(p+1)}\,, \qquad \hat{R}_{(4)} = 0\,.
\tag{4.15}
$$

4.3 Global monopole-like defect

The above procedure may not directly apply to $n \geq 3$, because for $n = 1$ and $n = 2$ the extra space is either flat or conformally flat, but for $n \geq 3$ the extra space is curved and hence the defect solution essentially introduces a solid angle deficit in the extra dimensions. The case is true even if one includes the higher-curvature terms into the field equations and assume $\hat{R}_{(4)} = 0$. As gestured in [25] the location of such defect in the brane might act as window in the higher dimensions. In the conventional Kaluza-Klein theory, our universe in higher dimensions has the topology of $M_4 \otimes K$, where K is the compact manifold and the isometries of K can be seen as gauge symmetries of the effective four-dimensional theory [26] and the 5-dimensional space-time manifold is factorized as $M_4 \otimes S^1$. However, in the brane set up with a non-factorizable geometry since the role of the gauge fields is played by the extra components of the graviton [6], the gauge fields may get mass when at least part of the K isometries spontaneously break down in the extra dimensions by the brane. This effect can be observed as the Higgs phenomena in the usual

$(3+1)$-dimensional world [25]. This may suggest that a hedge hog type scalar field may explain the monopole defect in the extra dimensions. This is indeed the case as we see in ref. [14] and also qualitatively discussed in [25].

So one can consider the case of $n \geq 3$ with the radial hedge hodge type solution as in ref. [14] which characterizes a global monopole defect in the extra dimensions. With this in mind the source terms can be described by a multiplet of n scalar fields $\Phi^a = \upsilon y^a/y$, where υ is the VEV at the minimum of the n-sphere, y is the extra radial coordinate. Since $\rho_0(y) = \rho_y(y) = -(n-1)\upsilon^2/2y^2$ and $\rho_\theta(y) = -(n-3)\upsilon^2/2y^2$, the identity (3.11) implies that $V'(y) = -1/y$. This brings the non-trivial term e^{2V} to the simple form and hence one can solve the Einstein equations explicitly by making an appropriate ansatz for $\mathrm{U}(y)$. Since this paper mainly focuses on the dynamics with higher order curvature terms, an investigation of such defect solutions deserves as a topic for separate publication.

5. Higher derivative field equations

In this section we shall confine ourselves to the case where the geometry is Gauss-Bonnet type (i.e. $4\alpha = -\beta = 4\gamma$) with the vanishing $(3+1)$-dimensional cosmological constant. Therefore, our metric ansatz will be

$$ds^2 = e^{-2\,\mathrm{U}(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 + e^{-2V(y)}d\Omega^2_{n-1}\,. \tag{5.1}$$

Using the Gauss-Bonnet relation, one can rewrite X_{AB} from eq. (2.2) in the following form:

$$X_{AB} = 2\alpha\Big(Y_{AB} - \frac{1}{4}g_{AB}\mathcal{R}^2_{GB}\Big) = 2\alpha L_{AB}\,, \tag{5.2}$$

where Y_{AB} is expressed by

$$Y_{AB} = RR_{AB} - 2R_{ACBD}R^{CD} + R_{ACDE}R_B{}^{CDE} - 2R_A{}^C R_{BC}\,. \tag{5.3}$$

In 3+1 dimensions L_{AB} is the covariantly conserved Lanczos vector. With the metric ansatz (5.1) we evaluate the following quantities in the most general forms

$$\begin{aligned}
\mathcal{R}^2_{GB} = {} & (p-1)p(p+1)\Big[(p+2)(U')^4 - 4(U')^2U''\Big] + \\
& + 2p(p+1)(n-1)\{np + 2(p+1)(n-2)\}(U')^2(V')^2 + \\
& + 4p(p+1)(n-1)\Big[(p+1)(U')^3V' - 2U'U''V' - p(U')^2V''\Big] - \\
& - 4(n-1)(n-2)(p+1)\Bigg[\; U''(V')^2 + 2U'V'V'' - (n-1)U'(V')^3 + \\
& \qquad\qquad + \left\{\frac{(p+2)}{2}(U')^2 - U''\right\}e^{2V}\Bigg] + \\
& + 2(n-1)(n-2)(n-3)\Big[2V'' - (n-2)(V')^2 - 2(p+1)U'V'\Big]e^{2V} +
\end{aligned}$$

$$
\begin{aligned}
& + (n-1)(n-2)(n-3)(n-4)e^{4V} \,, & (5.4)\\
Y_t{}^t =\; & (p-1)p\Big[(p+1)(U')^4 - 3(U')^2U''\Big] + \\
& + (n-1)\{(n-2)(3p+1)+2p\}(U')^2(V')^2 - \\
& - 4p(n-1)U'U''V' + \frac{p(n-1)}{2}\Big[2(3p+1)(U')^3V' - (p+1)(U')^2V''\Big] - \\
& - (n-1)(n-2)\Big[U''(V')^2 + (n-2)U'(V')^3 + \frac{(p+1)}{2}U'V'V''\Big] + \\
& + (n-1)(n-2)\Big[U'' - (p+1)(U')^2 - (n-3)U'V'\Big]e^{2V} \,, & (5.5)\\
Y_y{}^y =\; & (p-1)p(p+1)\Big[(U')^4 - (U')^2U''\Big] + (n-1)(p+1)(n+p-2)(U')^2(V')^2 + \\
& + p(p+1)(n-1)\Big[2(U')^3V' - 2U'U''V' - (U')^2V''\Big] + (n-1)(n-2)(p+1)\times \\
& \times \Big[U'(V')^3 - 2U'V'V'' - U''(V')^2 + \{U'' - (U')^2\}e^{2V}\Big] + (n-1)(n-2)\times \\
& \times (n-3)\Big[(V')^2V'' + (V')^4 + \{V'' + (V')^2\}e^{2V}\Big] \,, & (5.6)\\
Y_\theta{}^\theta =\; & p(p+1)\Big[(p+1)(U')^3V' - (U')^2V'' - 2U'U''V'\Big] + (p+1)(n-2)\times \\
& \times \Big[(3n-5)U'(V')^3 - 2U''(V')^2 - 4U'V'V'' + \{2U'' - (p+2)(U')^2\}e^{2V}\Big] + \\
& + (p+1)\{p(n-1) + 2(n-2)(p+1)\}(U')^2(V')^2 + (n-2)(n-3)\times & (5.7)\\
& \times\Big[\{3V'' - (2n-5)(V')^2 - 3(p+1)U'V'\}e^{2V} - 3V''(V')^2 + (n-1)(V')^4\Big] \,.
\end{aligned}
$$

5.1 Planar wall solution

In this subsection, we solve the higher derivative field equations for $n = 1$. These are the generalization of the well-known domain wall solution but includes contribution of higher-curvature terms into the field equations. Since the brane that we are considering is characterized by a conformally flat metric, we shall call the brane object a planar wall (i.e. $\hat{R}_{(4)} = 0$) embedded in the bulk AdS_5. With the metric (5.1), the field equations (2.2) reduce to the form

$$
\begin{aligned}
\frac{p(p+1)}{2}u^2 - \frac{(p-2)(p-1)p(p+1)}{2}\alpha\kappa^2u^4 &= -\kappa^2(\Lambda_b - \rho_0(y)) \,, & (5.8)\\
\frac{p(p+1)}{2}u^2 - \frac{(p-2)(p-1)p(p+1)}{2}\alpha\kappa^2u^4 &= -\kappa^2(\Lambda_b - \rho_y(y)) \,. & (5.9)
\end{aligned}
$$

Clearly we see that the planar wall solution, i.e. $n = 1$ and $\hat{R_{(4)}} = 0$, requires the constant source terms in the extra dimensions, i.e. $\rho_o(y) = \rho_y(y)$, and we obtain

$$
u^2 = \frac{1 \pm \sqrt{1 + \frac{8(p-1)(p-2)}{p(p+1)}\alpha\kappa^4(\Lambda_b - \rho_y)}}{2(p-1)(p-2)\alpha\kappa^2} \tag{5.10}
$$

whose metric solution is given by (4.4). For the 3-brane both the positive and negative root solutions are allowed if the inequality $0 \le \alpha\kappa^4(\rho_y - \Lambda_b) \le 3/4$ holds.

This also does not rule out the positive bulk cosmological constant. Clearly, unlike the case in domain wall solution, where $\rho_0 = \Lambda_b$ implies both $\hat{R}_{(4)} = 0$ and $u = 0$, the planar wall solution corresponds to $u = 0$ *or* $M_{(5)}^3/16\pi\alpha$. We remind the reader that the case $\alpha = 0$ does not correspond to the Einstein gravity as long as the 3-brane we are considering remains flat.

5.2 Planar string solution

Now we turn to the case of co-dimensions two, i.e. $n = 2$. With *ansatzs* (3.5) and (5.1) the field equations (2.2) then reduce to the form

$$\begin{aligned} -\Big(1 - \frac{a(p+1)}{2(p-1)}u^2\Big)V'' + (1 - au^2)\{(V')^2 + puV'\} + & \\ + \frac{p(p+1)}{2}u^2 - \frac{(p-2)(p+1)}{4}au^4 &= -\kappa^2(\Lambda_b - \rho_0)\,, \quad (5.11) \\ (p+1)(1-au^2)uV' + \frac{p(p+1)}{2}u^2 - \frac{(p-2)(p+1)}{4}au^4 &= -\kappa^2(\Lambda_b - \rho_y)\,, \quad (5.12) \\ \frac{(p+1)(p+2)}{2}u^2 - \frac{(p+1)(p+2)}{4}au^4 &= -\kappa^2(\Lambda_b - \rho_\theta)\,, \quad (5.13) \end{aligned}$$

where $a = 2p(p-1)\alpha\kappa^2$. We solve these equations for the physically interesting case of 3-brane and obtain, whose general metric solution is given by (4.12) with $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$, the following coefficients:

$$\begin{aligned} u^2 &= \frac{1 \pm \sqrt{1 + a\kappa^2(\Lambda_b - \rho_\theta)/5}}{a}\,, \\ V(y) &= \frac{uy}{b} + \frac{\alpha\kappa^4 u}{10}\int \frac{(4\Lambda_b - 5\rho_y + \rho_\theta)}{b(1+b)}dy\,, \end{aligned} \quad (5.14)$$

where $b = \sqrt{1 + a\kappa^2(\Lambda_b - \rho_\theta)/5}$ and $a = 12\alpha\kappa^2$. The identity derived from the equations. (5.11)–(5.13) turns out to be

$$b\{(V')^2 - V'' - U'V'\} + \sqrt{5}\kappa^2(\rho_0 - \rho_y) = 0\,. \quad (5.15)$$

For simplicity we choose the positive root of u^2. For the specific choice as $\Lambda_b = \rho_y = \rho_\theta$, i.e. either the sources are trivial along with $\Lambda_b = 0$ or they are the constant (i.e. $\rho_y = \rho_\theta = \Lambda_b$) in the extra dimensions, the general metric solution is given by (4.14), and just for the trivial sources ($\rho_i = 0$) one has

$$ds^2 = e^{-2u|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 + r_0^2 e^{-2u_0|y|b^{-1}}d\theta^2\,, \quad (5.16)$$

where the additional warp factor b^{-1} is given by $b = \sqrt{1 + 12\alpha\kappa^4\Lambda_b/5}$. Either with the vanishing bulk cosmological constant or the specific tuning condition, i.e. $\Lambda_b = \rho_\theta$, one gets $b = 1$ and hence the metric solution again reduces to (4.14).

6. Dilatonic Gauss-Bonnet gravity

6.1 Dilaton in the brane background

In this section we consider the string inspired higher derivative gravity action in the dilatonic Gauss-Bonnet combination and study its compatibility with the RS type non-factorizable metrics (5.1) and the string amplitude computations, in particular the conformal weights of the dilaton couplings to the higher-curvature terms [21]. The above mentioned action in the low energy limit, by rescaling the dilaton to get standard normalization of the propagator [21], may be written in $D(= d+1)$-dimensional space-time as

$$S_M = \int d^{d+1}x\sqrt{-g}\Big\{R - \epsilon(y)e^{m\Phi} + \lambda' e^{n\Phi}\alpha(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}) - \\ - \frac{4}{d-1}(\nabla\Phi)^2 - V(\Phi) + \cdots\Big\} + \int_{\partial B} d^d x\sqrt{-g_{\rm brane}}e^{q\Phi(y)}\tilde{\sigma}\,, \quad (6.1)$$

where the last term is the contribution of boundary action defined at the position of p-brane, $\lambda' = \alpha'/8g_s^2 > 0$, α' is the Regge slope, g_s is the string coupling constant, $\tilde{\sigma}$ is the brane tension, $V(\Phi)$ is the dilatonic potential, which and the subsequent terms denoted by dots we set zero in the present consideration.[1] For generality one can keep the conformal parameters m, n, q arbitary in the higher derivative gravity theory coupled to the dilaton. Nevertheless, m and n are fixed for the particular space-time dimensions by matching the coefficients with the tree-level amplitudes to $\mathcal{O}(\alpha')$ when one restricts the action (6.1) to the effective string amplitudes computations. In the lower dimensions, $d < 10$, the term $\epsilon(y)$ may be realized as the negative of the bulk cosmological constant. So we restrict the action (6.1) in the $d+1 = 5$-dimensional space-time. As explored in [27] the RS type brane solutions can not be obtained in the low energy supergravities coming from string theory. A possibility left open in [27] that the higher order curvature corrections in the string theory should allow brane world solutions can be realized if one defines $v(y) = -\Lambda_b$. In this sense the term $\epsilon(y)$ is crucial in the obtention of RS type solutions.

The graviton equation of motion derived from the action (6.1) can be expressed in the form, defining $e^{n\Phi(y)} = f(\Phi)$,

$$0 = G_{AB} - \frac{1}{2}\alpha\lambda' f(\Phi)(g_{AB}\mathcal{R}^2_{GB} - 4Y_{AB}) - \frac{4}{3}\Big(\nabla_A\Phi\nabla_B\Phi - \frac{1}{2}g_{AB}(\nabla\Phi)^2\Big) + \\ + 2\alpha\lambda'\Big\{R(g_{AB}\nabla^2 - \nabla_A\nabla_B)f(\Phi) - 2g_{AB}\nabla^C\nabla^D f(\Phi)R_{CD} - 2\nabla^2 f(\Phi)R_{AB} - \\ - 4\nabla_C f(\Phi)\nabla^C R_{AB} + 4\nabla_{(A}\nabla_{|C|}f(\Phi)R_{B)}{}^C +$$

[1] When this paper was being typed the reference [24] with similar effective action appeared in the lanl e-Print archive where the form of potential $-V(\Phi) = c_2 f(\Phi)(\nabla_A\Phi)^4 + \cdots$ is also included

$$
\begin{aligned}
&+4\nabla_C f(\Phi)\nabla_{(A}R_{B)}{}^C + 2\nabla^{(C}\nabla^{D)}f(\Phi)R_{ACBD}\Big\} + \\
&+\frac{1}{2}g_{AB}\epsilon(\Phi)e^{m\Phi} - e^{q\Phi(y)}\frac{\sqrt{-g_b}}{2\sqrt{-g}}\delta^\mu_A\delta^\nu_B(g_b)_{\mu\nu}\sigma\,, \qquad (6.2)
\end{aligned}
$$

while the dilaton equation of motion is given by

$$
0 = \alpha\lambda' f'(\Phi)\mathcal{R}^2_{GB} + \frac{8}{3}\nabla^2\Phi - m\epsilon(y)e^{m\Phi} + qe^{q\Phi}\frac{\sqrt{-g_b}}{\sqrt{-g}}\delta^\mu_A\delta^\nu_B(g_b)_{\mu\nu}\sigma\,. \qquad (6.3)
$$

In eq. (6.3) prime denotes the differentiation w.r. to Φ, $\mathcal{R}^2_{GB}$ is the Gauss-Bonnet term, Y_{AB} was defined previously in the section 5 and $\sigma = \tilde{\sigma}\delta(y)$. With the metric ansatz (5.1), for $d+1 = 5$, the field equations (6.2) and (6.3) reduce to the following forms:

$$
\begin{aligned}
12\lambda'\alpha e^{n\Phi}\{(U')^4 - (U')^2U'' - 3n(U')^3U'' + n^2(U')^2(\Phi')^2 + 2nU'U''\Phi' + \\
+ n(U')^2\Phi''\} - 6(U')^2 + 3U'' - \frac{2}{3}(\Phi')^2 - \frac{1}{2}\epsilon(y)e^{m\Phi} + \frac{1}{2}e^{q\Phi}\sigma = 0\,, \qquad (6.4)\\
12\lambda'\alpha e^{n\Phi}\{(U')^4 - 4n(U')^3\Phi'\} - 6(U')^2 + \frac{2}{3}(\Phi')^2 - \frac{1}{2}\epsilon(y)e^{m\Phi} = 0\,, \qquad (6.5)\\
24n\lambda'\alpha e^{n\Phi}\{5(U')^4 - 4(U')^2U''\} + \frac{8}{3}\Phi'' - \frac{32}{3}U'\Phi' - m\epsilon(y)e^{m\Phi} + qe^{q\Phi}\sigma(y) = 0\,. \qquad (6.6)
\end{aligned}
$$

Here the primes denote differentiation w.r. to the fifth bulk coordinate(y). Notice that for the case of constant dilaton field w.r.t. y (i.e. $\Phi(y) = \Phi_0, \Phi'(y) = \Phi''(y) = 0$) and the bulk geometry having a warp factor $\mathrm{U}(y) = u|y|$, where u is some constant function, the first two equations in the bulk reduce to a single equation, i.e.

$$
-12u^2 + 24\lambda'\alpha e^{n\Phi_0}u^4 - \epsilon(y)e^{m\Phi_0} = 0\,. \qquad (6.7)
$$

This implies that in the bulk because of the Bianchi identities there could only be two independent equations. To see this explicitly from the equations (6.4)–(6.6) one can formally arrive at the following identity (see also [24])

$$
e^{q\Phi(y)}\sigma(y)\{4U'(y) - q\Phi'(y)\} - e^{m\Phi}\epsilon'(y) = 0\,. \qquad (6.8)
$$

When one defines $\epsilon(y)(= -\Lambda_b)$ as the bulk cosmological constant, then the requirement that $\epsilon'(y) = 0$ equally avoids the breaking of Poincaré invariance in the bulk [4, 5, 24].

6.2 Constant dilaton solution

One can solve the above field equations with the constant dilaton in the bulk, i.e. the dilaton remains constant w.r. to the radial bulk coordinate, $\Phi(y) = \Phi_0$. This can also be associated with the D3-brane solution of certain version of the string theories. For this case, the Bianchi identity (6.8) is reduced to

$$
4e^{q\Phi_0}U'(y)\tilde{\sigma}\delta(y) = 0\,, \qquad (6.9)
$$

which is trivial since $\delta(y) = 0$ in the bulk. And the field equations (6.4) and (6.5) reduce to the following forms:

$$\frac{d}{dy}(8\lambda'\alpha(U')^3 e^{n\Phi_0} - 6U') = e^{q\Phi_0}\tilde{\sigma}\delta(y)\,, \tag{6.10}$$

$$\epsilon(y)e^{m\Phi_0} + 12(U')^2 - 24\lambda'\alpha(U')^4 e^{n\Phi_0} = 0\,, \tag{6.11}$$

where in obtaining eq. (6.10) we have utilized the equation (6.11). In the bulk equation (6.10) implies that

$$8\lambda'\alpha(U')^3 e^{n\Phi_0} - 6U' = c\,, \tag{6.12}$$

where c is an integration constant. For the constant dilaton the additional terms $-V(\Phi) = c_2 f(\Phi)(\nabla_A(\Phi))^4 \cdots$ considered in [24] does not affect this equation. As a third-degree equation, (6.12) must have atleast one real solution for the arbitary constant c. Indeed, it gives all three real solutions if we impose $a \leq c/2 \leq -a$, where $a = e^{-n\Phi_0/2}/\sqrt{\lambda'\alpha}$. However, to make the various conformal parameters: m, n, q compatible with the string amplitude computations, one needs to restrictively define the integration constant as $c = 8e^{-n\Phi_0/2}/3\sqrt{3\lambda'\alpha}$ with some suitable junction conditions about the brane position. With this one of the real solutions of eq. (6.10) is given by

$$U'(y) = k_+ = -k_- = k = \frac{e^{-n\Phi_0/2}}{2\sqrt{3\lambda'\alpha}}\,. \tag{6.13}$$

Obviously, if $y > 0$ corresponds to the solution $U'(y) = k_+$, then $y < 0$ corresponds to $U'(y) = -k_-$. The above choice of c uniquely reproduces the conformal weight of dilaton couplings compatible with string amplitude computations. If one likes, this can also be related to the RS solution. For the vanishing dilaton field this implies $k = k_+ = -k_- = 1/(2\sqrt{3\lambda'\alpha})$. Ofcourse, the solution with $k_+ = k_-$ can not be the R-S solution as this has continuous metric function at $y = 0$ which violates the symmetry about the brane position. Before going to make any conclusion here, we first give the full treatment of all equations both in the bulk and on the brane and find the general and consistent solutions.

Again, integrating the equation (6.10) over an interval that includes the brane position at $y = 0$, one gets

$$e^{q\Phi_0}\tilde{\sigma} = -6(k_+ - k_-) + 8\lambda'\alpha(k_+^3 - k_-^3)e^{n\Phi_0}\,. \tag{6.14}$$

This relates the $k_+,\ k_-$ with the brane tension $\tilde{\sigma}$. Now, one needs to solve eq. (6.9) with the continuity of bulk cosmological constant $\Lambda_b'(y) = -\epsilon'(y)$ at the brane position $y = 0$. Therefore, from (6.11) one has

$$\epsilon(y)e^{m\Phi_0} = 12k_\pm^2(2\lambda'\alpha e^{n\Phi_0} - 1)\,. \tag{6.15}$$

This implies

$$\epsilon(y)e^{m\Phi_0} = 12k_+^2(2\lambda'\alpha k_+^2 e^{n\Phi_0} - 1) = 12k_-^2(2\lambda'\alpha k_-^2 e^{n\Phi_0} - 1)\,. \tag{6.16}$$

This equation is solved for (i) $k_+ = -k_- = k$ and (ii) $k_+^2 + k_-^2 = e^{-n\Phi_0}/2\lambda'\alpha$. Notice that for the constant $c = 2e^{-n\Phi_0/2}/\sqrt{\lambda'\alpha}$, eq. (6.10) can have two real solutions as $k_\pm = \pm e^{-n\Phi_0/2}/\sqrt{\lambda'\alpha}$ and $k_\pm = \pm e^{-n\Phi_0/2}/2\sqrt{\lambda'\alpha}$ which satisfy both the solutions of (6.16) independently. If this is the case, the theory is compatible only with RS type space-time and but not with the string amplitudes computation. The solution which is compatible with both the RS type geometry and the string theory is given by (6.13). And, a possible pair of imaginary solutions is discarded since this demands $k_+^2 + k_-^2 < 0$ and hence $\lambda' < 0$ for positive α, but this is again not compatible with string amplitude computations. Therefore, for the choice $k_+ = -k_- = k$, one gets

$$\epsilon(y) = 12k^2(2\lambda'\alpha k^2 e^{n\Phi_o} - 1)e^{-m\Phi_0}\,. \tag{6.17}$$

Now we solve eq. (6.6) in the bulk, where one can set $\Phi'(y) = \Phi''(y) = 0$, but proper care should be taken at the brane position, i.e. $y = 0$ where discontinuity of the function may appear due to the delta function source. Equation (6.6), in conjuction with eq. (6.11) and $U''(y) = 0$, reduces to the form

$$(5n - m)\epsilon(y)e^{m\Phi_0} + 60nk_\pm^2 + qe^{q\Phi_0}\tilde\sigma(y)\delta(y) = 0 \tag{6.18}$$

which, in the bulk, on using (6.15) reduces to the form

$$2(5n - m)\lambda'\alpha k_\pm^2 e^{n\Phi_0} + m = 0\,. \tag{6.19}$$

For the solution $k_+ = -k_- = k$ one has

$$m = \frac{10n\lambda'\alpha k^2 e^{n\Phi_0}}{2\lambda'\alpha k^2 e^{n\Phi_0} - 1}\,. \tag{6.20}$$

If we integrate eq. (6.6) in the neighbourhood of brane position, $y = 0$, with $U'(y) = k_+ = -k_-$, one is left with

$$\begin{aligned} q\int_{0_-}^{0_+} e^{q\Phi_0}\tilde\sigma\delta(y) &= \int_{0_-}^{0_+} \frac{d}{dy}(32n\lambda'\alpha e^{q\Phi_0}(U'(y))^3)dy\,, \\ \text{i.e. } qe^{q\Phi_0}\tilde\sigma &= 32n\lambda'\alpha e^{n\Phi_0}(k_+^3 - k_-^3)\,. \end{aligned} \tag{6.21}$$

Equation (6.21) in conjuction with eq. (6.14) implies

$$\tilde\sigma = \frac{48nk}{q - 4n}e^{-q\Phi_0}\,, \tag{6.22}$$

where

$$q = \frac{16nk^2\lambda'\alpha}{4k^2\lambda'\alpha e^{n\Phi_0} - 3}\,. \tag{6.23}$$

Notice that in the string amplitude computations [22, 21], the conformal parameters m and n are fixed for the given space-time dimensions of the string frame. In $d+1=5$ dimensions, $m = 4/\sqrt{3(d-1)} = 4/3 = -n$. With these conformal weights and vanishing dilaton field, from eqs. (6.20), (6.23), (6.22) and (6.17) we get (see also [24])

$$k = \frac{1}{2\sqrt{3\lambda'\alpha}}\,, \qquad q = \frac{2}{3}\,, \qquad \tilde{\sigma}(y) = -\frac{32k}{3}\,, \qquad \Lambda_b = -\epsilon(y) = 10k^2\,, \tag{6.24}$$

where $\lambda' = 1/8g_s^2$. These values are well consistent with both the R-S brane world scenario and 5-dimensional string theory amplitudes. In the former case the bulk cosmological constant $\Lambda_b(=10k^2)$ appears to have opposite sign than that of the brane tension $\tilde{\sigma}(= -32k/3)$ to ensures the positive tension brane at $y=0$, and in the latter case the required values and relations among the conformal weights of dilaton coupling in the 5-dimensional string theory are satisfied with the proper choice of λ' or α or both.

A possible solution of eq. (6.12) as

$$k_+ = -k_- = k = \frac{e^{-n\Phi_0/2}}{2\sqrt{\lambda'\alpha}}\,, \tag{6.25}$$

can be associated to the RS solution. This also reproduces the correct sign of conformal weight of the dilaton couplings as in the string amplitude computations. One has in this case

$$m = -5n = \frac{4}{3}\,, \qquad q = \frac{8}{15}\,, \qquad \tilde{\sigma} = -8k\,, \qquad \Lambda_b = 6k^2\,. \tag{6.26}$$

Further with $k_+ = -k_- = e^{-n\Phi_0/2}/\sqrt{\lambda'\alpha}$, one gets $m=10$, $n=4/3$, $q=32/15$, $\tilde{\sigma} = 4k$ and $\Lambda_b = -12k^2$.

Finally, for the solution $k_+^2 + k_-^2 = e^{-n\Phi_0}/2\sqrt{\lambda'\alpha}$, but $k_+ \neq -k_-$, one has from eq. (6.19) that $m + 5n = 0$ and $m(1 - 4\lambda'\alpha e^{n\Phi_0}k_\pm^2) = 0$. This implies that $m = -5n = 0$, which from eq. (6.21) further implies that $q = 0$. For the trivial dilaton couplings these results somewhat contradict to those of ref. [20], where there exists some dependence of $k_\pm$ on $\epsilon(y)(= -\Lambda_b)$ for non-trivial α and λ'. Similar conclusions have recently been made in [24] and the solutions for linear dilaton with the trivial case $m = n = q = 0$ is also generalized. However, there are no solutions of the bulk equation (6.12) other than mentioned above but still holds the equality $k_+^2 + k_-^2 = e^{-n\Phi_0}/2\sqrt{\lambda'\alpha}$. Hence, we are in a dilema if the two solutions of (6.16) are linearly independent. The case of linear dilaton with $m = n = q = 0$, $V(\Phi) = 0$ and $\epsilon(y) = \mathcal{V}(\Phi)$, dilaton super potential is studied in [19].

6.3 Linear dilaton: time dependent solution

Our starting point for this subsection is also the string-inspired low energy effective action in the higher derivative model without torsion However, unlike the previous subsection, here we assume a simple dilaton-gravity coupling also with the curvature scalar (R) and confine ourselves to the 5-dimensional spacetime. Thus,

$$S_M = \int d^5x\sqrt{-g}e^{-2\Phi}\Big\{R + \lambda'\alpha(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}) - \\ - 4(\nabla\Phi)^2 - V(\Phi)\Big\}, \tag{6.27}$$

where $V(\Phi)$ is the dilatonic potential which we again set zero. The inflationary solutions to the above action by considering a conformally flat Friedmann-Robertson-Walker metric were realized in [23], where the extra dimensions are equally treated as large as the usual 3 spatial dimensions. But this is not the case we are considering. We assume the background metric in the following form:

$$ds^2 = -d\tau^2 + e^{-2U(\tau,y)}\delta_{ij}dx^i dx^j + e^{-2W(\tau,y)}(dr^2 + r^2 d\Omega_{n-1}^2)\,, \tag{6.28}$$

where $d\tau$ is the conformal time interval, so that the usual $(3+1)$-dimensional metric can be expressed in the form $e^{-2U(t,y)}\eta_{\mu\nu}dx^\mu dx^\nu$, the indices i,j denote the spatial indices $1,2,3$. We solve the field equations in $D = 5$-dimensional space-time with a specific choice of the ansatz $U^{-1}(\tau,y) = W(\tau,y)$. This choice may be realized for the inflationary solutions with the expanding external space and contracting internal spatial dimensions, which should also be true in brane set up where as the brane inflates the extra bulk coordinates get contracted. However, this may not be the case with stable non-compact extra dimensions. We concentrate only on the case with $n = 1$. Assuming the usual 3-dimensional space as homogeneous and isotropic, the 5-dimensional effective action reduces to the form:

$$S_M = \int d\tau e^{-2\Phi}e^{-2U}\Big\{2H^2 + \dot{H} + \dot{\Phi}^2 - 6\lambda'\alpha(2\dot{H}H^2 + H^4)\Big\}\,, \tag{6.29}$$

where dot denotes the differentiation $\partial/\partial\tau$ and we have defined $-\dot{U}(\tau) = H(\tau)$, the Hubble parameter. The corresponding field equations for the scale factor and dilaton field are

$$\ddot{\Phi} - \dot{\Phi}^2 + 2H^2 + \dot{H} + 2H\dot{\Phi} - 6\lambda'\alpha(2\dot{H}H^2 + H^4) = 0\,, \tag{6.30}$$

$$\ddot{\Phi} - 3\dot{\Phi}^2 + 6\lambda'\alpha\{H^4 + 2H^2\dot{H} - 4\dot{\Phi}H^3 - 4H\dot{H}\dot{\Phi} - 2H^2(\ddot{\Phi} - 2\dot{\Phi}^2)\} = 0\,. \tag{6.31}$$

Indeed, one can modify the action (6.24) by adding a boundary term so that it does not contain the second derivative of H. However, here we solve the above two equations for a linear dilaton field with the constant Hubble parameter, i.e. $\dot{\Phi} = \upsilon =$ constant, $H = H_0 =$ constant. and obtain the following exact real solution: (i) the trivial solution corresponds to $H_0 = 0$, $\dot{\Phi} = 0$, and (ii) the non-trivial real solution implies

$$H_0 = \frac{1}{2\sqrt{2\lambda'\alpha}}, \frac{1}{\sqrt{2\lambda'\alpha}}\,, \tag{6.32}$$

$$\upsilon = \Big\{1 + \sqrt{3(1 - 2\lambda'\alpha H_0^2)}\Big\}H_0\,. \tag{6.33}$$

There could be a pair of imaginary solutions both to H_0 and υ, but we discard them. The above solutions could be physically more meaningful as compared to the solutions found in [23], in the sense that one needs either of the coupling constants λ' or γ_3 (see the ref. [23]) to be negative. If one judges the action from string low energy effective action and the required relations among Gauss-Bonnet couplings α, β, γ, a somewhat unphysical relation persists, otherwise in their solution the Hubble parameter becomes imaginary but this is not the case in our solution. Further, the metric ansatz chosen in [23] is conformally flat FRW type and is somewhat unphysical to explain the inflationary solutions, in the sense that the extra spatial dimensions equally inflate during the inflationary epoch. But in our parametrization, the extra spatial dimensions get contracted when the usual 3-brane world volume expands.

7. Discussions and outlook

The background metric with the Ricci constant curvature would imply a constraint equation in the theory of higher derivative gravity which defines the bulk space-time geometry as AdS(dS). We have studied various solutions of the higher derivative gravity in the brane background by considering the string inspired effective actions in the Gauss-Bonnet combination. The defect solutions in higher dimensions known to the Einstein equations are generalized by including the contribution of higher-curvature terms into the field equations and the general metric solutions that correspond to planar defects are obtained. The string inspired higher derivative gravity theory in the dilatonic Gauss-Bonnet combination is shown to be consistent to the RS type non-factorizable geometry and the conformal weights of the dilatonic couplings in the string amplitude computations with the proper choice of free parameters of the theory. Discussed is also time dependent dilaton solutions in some version of string inspired higher derivative gravity theory.

There are few interesting topics which deserve future investigations. One of these is to extend the sections 5 and 6 to explain the different brane-world black-hole (BH) solutions without and with dilaton. One such solution as Schwarzscild AdS_5 black-hole in the background of constant Ricci curvature space-time is realized in ref. [28]. However, it is also important to further explore other BH solutions, with more general metric ansatz in the curved and flat brane background. It is also interesting to investigate the cosmological implications of the higher derivative theory of the type considered in section 6.1–6.2.

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